2035. *Proposed by Gregory Dresden, Prakriti Panthi (student), Anukriti Shrestha (student) and Jiahao Zhang (student), Washington & Lee University, Lexington, VA.*

Two real numbers x, y are said to *have a common decimal part* if $xy < 0$ and $x + y$ is an integer, or else $xy \geq 0$ and $x - y$ is an integer. More concretely, this means that the decimal expansions of x, y are of the forms

$$\pm a_m a_{m-1} \ldots a_1 a_0 . d_1 d_2 d_3 \ldots,$$
$$\pm b_n b_{n-1} \ldots b_1 b_0 . d_1 d_2 d_3 \ldots,$$

where the common decimal part is $0.d_1 d_2 d_3 \ldots$.

Find all polynomials of degree at least 2 with integer coefficients, all roots real, and irreducible over the rationals, whose roots have pairwise common decimal tails.