PROBLEMS

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Proposals

To be considered for publication, solutions should be received by May 1, 2018.

2031. *Proposed by Barış Burçin Demir, Ankara, Turkey.*

Let n be an integer, $n \geq 2$. Let $A_1A_2A_3 \cdots A_{2n+1}$ be a regular polygon with $2n+1$ sides. Let P be the intersection of the segments $\overline{A_2A_{n+2}}$ and $\overline{A_3A_{n+3}}$. Prove that

$$(A_1P)^2 = (A_2A_3)^2 + (A_3P)^2.$$

2032. *Proposed by Noah H. Rhee, University of Missouri–Kansas City, MO.*

Let a, b be real numbers with $a < b$, and let f be a continuous, strictly increasing function on the closed interval $[a, b]$. For $y \in \mathbb{R}$, define

$$E(y) = \int_a^b |f(x) - y|\, dx.$$

Prove that $E(y)$ has a minimum value as y varies in $\mathbb{R}$, and find all y for which the minimum is attained.

2033. *Proposed by Yoshihiro Tanaka, Hokkaido University, Sapporo, Japan.*

A deck is the collection of all 52 pairs ("cards") of the form (n, s) where $1 \leq n \leq 13$ is the number on the card, and the suit s of the card is one of the symbols $\diamondsuit$, $\heartsuit$, $\spadesuit$, $\clubsuit$. Given an arbitrary partition of a deck into 13 sets $S_1, S_2, \ldots, S_{13}$ of 4 cards each, prove that there exists a corresponding partition C_1, C_2, C_3, C_4 of the deck into 4 sets of 13 cards each, such that each of the parts C_i $(1 \leq i \leq 4)$ satisfies:

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We invite readers to submit original problems appealing to students and teachers of advanced undergraduate mathematics. Proposals must always be accompanied by a solution and any relevant bibliographical information that will assist the editors and referees. A problem submitted as a Quickie should have an unexpected, succinct solution. Submitted problems should not be under consideration for publication elsewhere.

Proposals and solutions should be written in a style appropriate for this Magazine.

Authors of proposals and solutions should send their contributions using the Magazine's submissions system hosted at http://mathematicsmagazine.submittable.com. *More detailed instructions are available there. We encourage submissions in PDF format, ideally accompanied by LaTeX source. General inquiries to the editors should be sent to* mathmagproblems@maa.org.

(i) C_i has one card from S_j for $1 \le j \le 13$, and
(ii) the cards in C_i all have different numbers.

2034. *Proposed by Julien Sorel, Piatra Neamt, PNI, Romania.*

Let $\mathfrak{C}$ be a circle. Two points A, B are independently chosen on the circumference of $\mathfrak{C}$, uniformly at random. Two further points C, D are independently chosen in the interior of $\mathfrak{C}$ uniformly at random. What is the probability that D shall lie inside $\triangle ABC$?

2035. *Proposed by Gregory Dresden, Prakriti Panthi (student), Anukriti Shrestha (student) and Jiahao Zhang (student), Washington & Lee University, Lexington, VA.*

Two real numbers x, y are said to *have a common decimal part* if $xy < 0$ and $x + y$ is an integer, or else $xy \ge 0$ and $x - y$ is an integer. More concretely, this means that the decimal expansions of x, y are of the forms

$$\pm a_m a_{m-1} \ldots a_1 a_0 \,.\, d_1 d_2 d_3 \ldots,$$
$$\pm b_n b_{n-1} \ldots b_1 b_0 \,.\, d_1 d_2 d_3 \ldots,$$

where the common decimal part is $0.d_1 d_2 d_3 \ldots$.

Find all polynomials of degree at least 2 with integer coefficients, all roots real, and irreducible over the rationals, whose roots have pairwise common decimal tails.

Quickies

1075. *Proposed by Raymond Mortini, Université de Lorraine and IECL, France.*

Let $f : \mathbb{R} \to \mathbb{R}$ be a bounded and continuous function. Assume that there exist $a, b \in \mathbb{R}$ such that $f(a) \le f(x) \le f(b)$ for all $x \in \mathbb{R}$. Is it true that, for every $d > 0$, there exists a horizontal segment of length d with endpoints on the graph of f?

1076. *Proposed by Lokman Gökçe, Adana, Turkey.*

A quadrilateral $ABCD$ has angles $\angle ABC = 138^\circ$, $\angle BAD = 108^\circ$ and sides $AB = AD = \sqrt{3}\,BC$. What are the measures of angles $\angle ADC$ and $\angle BCD$?

Solutions

Consecutive successes in independent Bernoulli trials **October 2016**

2001. *Proposed by Herb Bailey and Dianne Evans, Rose-Hulman Institute of Technology, IN.*

Fix positive integers n and k. Numbers are drawn one at a time with replacement from an urn containing one of each of the first n positive integers. Find the expected number of drawings needed until k successive drawings are all ones.

Solution by Nicholas C. Singer, Annandale, VA.
Let E_k be the expected number of drawings. We claim that

$$E_k = \sum_{i=1}^{k} n^k = \begin{cases} k & (n = 1) \\ \frac{n^{k+1}-n}{n-1} & (n > 1). \end{cases} \tag{1}$$