PROBLEMS AND SOLUTIONS

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This section contains problems intended to challenge students and teachers of college mathematics. We urge you to participate actively *both* by submitting solutions and by proposing problems that are new and interesting. To promote variety, the editors welcome problem proposals that span the entire undergraduate curriculum.

Proposed problems should be sent to **Greg Oman**, either by email (preferred) as a pdf, TEX, or Word attachment or by mail to the address provided above. Whenever possible, a proposed problem should be accompanied by a solution, appropriate references, and any other material that would be helpful to the editors. Proposers should submit problems only if the proposed problem is not under consideration by another journal.

Solutions to the problems in this issue should be sent to **Chip Curtis**, either by email as a pdf, TEX, or Word attachment (preferred) or by mail to the address provided above, no later than May 15, 2021. Sending both pdf and TEX files is ideal.

PROBLEMS

1186. *Proposed by Gregory Dresden, Washington and Lee University, Lexington, VA and ZhenShu Luan (high school student), St. George's School, Vancouver, BC, Canada.*

Find a closed-form expression for the continued fraction $[1, 1, \ldots, 1, 3, 1, 1, \ldots, 1]$, which has n ones before, and after, the middle three.

1187. *Proposed by Reza Farhadian, Lorestan University, Khorramabad, Iran.*

Let $\alpha > 1$ be a fixed real number, and consider the function $M\colon [1, \infty) \to \mathbb{N}$ defined by $M(x) = \max\{m \in \mathbb{N}\colon m! \leq \alpha^x\}$. Prove the following:

$$\lim_{n\to\infty} \frac{\sqrt[n]{M(1)M(2)\cdots M(n)}}{M(n)} = e^{-1}.$$

1188. *Proposed by Ángel Plaza, Universidad de Las Palmas de Gran Canaria, Las Palmas de Gran Canaria, Spain.*

Let $\{f_n(x)\}_{n\geq 1}$ be the sequence of functions recursively defined by $f_n(x) = \int_0^{f_{n-1}(x)} \sin t\,dt$, with initial condition $f_1(x) = \int_0^x \sin t\,dt$. For each $n \in \mathbb{N}$, find the value of p_n such that $L_n = \lim\limits_{x\to 0} \frac{f_n(x)}{x^{p_n}} \in \mathbb{R}\backslash\{0\}$ and the corresponding value L_n. Prove also that $\log_2(L_n^{-1}) = 3\log_2(L_{n-1}^{-1}) - 2\log_2(L_{n-2}^{-1})$ for $n \geq 3$.

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