PROBLEMS

Click here to submit problems proposals as well as solutions, comments and generalizations to any problem in this section.

To facilitate their consideration, solutions should be received by **June 1, 2022**.

4721. *Proposed by Greg Dresden and Myanna Nash.*

It's well-known that if we take the sum along a diagonal in Pascal's triangle, like this,

$$\binom{n}{0} + \binom{n-1}{1} + \binom{n-2}{2} + \binom{n-3}{3} + \dots$$

then we get the Fibonacci numbers f_n which satisfy $f_n = f_{n-1} + f_{n-2}$.

But what if we take sums along diagonals at a more gentle slope? Let's define the numbers a_n as

$$a_n = \binom{n}{0} + \binom{n-1}{2} + \binom{n-2}{4} + \binom{n-3}{6} + \dots.$$

Prove that $a_n = a_{n-1} + a_{n-2} + a_{n-4}$.

4722. *Proposed by George Apostolopoulos.*

Let I be the incenter of triangle ABC and let A', B' and C' be the intersections of the rays AI, BI and CI with the circumcircle of the triangle. Prove that $\text{Area}(A'B'C') \geq \text{Area}(ABC)$.

4723. *Proposed by Michel Bataille.*

Let $(u_n)_{n\geq 1}$ be the sequence defined by $u_1 = 1$ and the recursion

$$u_{n+1} = u_n + 2^{2n-1} \cdot u_n^2.$$

Express u_n as a function of n.

4724. *Proposed by D.M. Bătineţu - Giurgiu and Daniel Sitaru.*

Find all $x, y > 0$ such that:

$$\frac{1}{(x+1)^8} + \frac{1}{(y+1)^8} = \frac{1}{8(xy+1)^4}$$