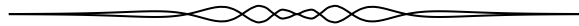


# PROBLEMS

*Click here to submit problems proposals as well as solutions, comments and generalizations to any problem in this section.*

To facilitate their consideration, solutions should be received by **June 1, 2022**.



**4721.** *Proposed by Greg Dresden and Myanna Nash.*

It's well-known that if we take the sum along a diagonal in Pascal's triangle, like this,

$$\binom{n}{0} + \binom{n-1}{1} + \binom{n-2}{2} + \binom{n-3}{3} + \dots$$

then we get the Fibonacci numbers  $f_n$  which satisfy  $f_n = f_{n-1} + f_{n-2}$ .

But what if we take sums along diagonals at a more gentle slope? Let's define the numbers  $a_n$  as

$$a_n = \binom{n}{0} + \binom{n-1}{2} + \binom{n-2}{4} + \binom{n-3}{6} + \dots$$

Prove that  $a_n = a_{n-1} + a_{n-2} + a_{n-4}$ .

**4722.** *Proposed by George Apostolopoulos.*

Let  $I$  be the incenter of triangle  $ABC$  and let  $A'$ ,  $B'$  and  $C'$  be the intersections of the rays  $AI$ ,  $BI$  and  $CI$  with the circumcircle of the triangle. Prove that  $\text{Area}(A'B'C') \geq \text{Area}(ABC)$ .

**4723.** *Proposed by Michel Bataille.*

Let  $(u_n)_{n \geq 1}$  be the sequence defined by  $u_1 = 1$  and the recursion

$$u_{n+1} = u_n + 2^{2n-1} \cdot u_n^2.$$

Express  $u_n$  as a function of  $n$ .

**4724.** *Proposed by D.M. Bătinețu - Giurgiu and Daniel Sitaru.*

Find all  $x, y > 0$  such that:

$$\frac{1}{(x+1)^8} + \frac{1}{(y+1)^8} = \frac{1}{8(xy+1)^4}$$